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Pressure Drop and Holdup in Stratified Gas-Liquid Flow

A mathematical model and an iterative procedure to calculate holdup and pressure drop in horizontal gas-liquid flow is developed. The predictions of the model agree with well over a hundred data points collected with air-water and air-glycerine solutions in 0.0254-, 0.0381-, and 0.0508-m. diameter pilot pipelines. A design procedure using the verified model is presented.

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SCOPE

Over the past three decades, pressure drop and holdup data have been collected for horizontal gas-liquid systems, and many attempts have been made to develop from the data general procedures for predicting these quantities. The current state of this art has been discussed in review articles by Anderson and Russell (1965, 1966), Simpson (1968), DeGance and Atherton (1970), and in texts by Govier and Aziz (1972), Scott (1964), and Wallis (1969). All came to essentially the same conclusion: No satisfactory general correlation exists. Errors of about 20 to 40% can be expected in holdup or pressure-drop prediction, and even this range is optimistic if one attempts to use the various predictive schemes without applying a generous measure of experience and judgment. A major difficulty in developing a general correlation based on statistical evaluation of data is deciding on a method of properly weighing the fit in each flow regime. It is difficult to decide, for instance, whether a correlation giving a good fit with annular flow and a poor fit with stratified flow is a better correlation than one giving a fair fit for both kinds of flow.

Although general correlations must continue to be used by those faced with two-phase flow problems, alternate procedures should be developed if we are to improve our ability to predict holdup and pressure drop. The complicated fluid motions in most two-phase flows of pragmatic interest make it impossible to develop the kinds of mathematical description that have proven so useful in single-phase flow analysis and design. Nevertheless, useful insights into the more complex problem are obtained by analyzing simple two-phase flow, as illustrated in this paper. In addition, this study provides a simple, tractable model of the flow and a clearcut, easy-to-use computational procedure for determining pressure drop and holdup. This analysis was also motivated by a need to establish a simple mathematical description for stability analysis to predict flow-pattern transition and has proven most useful in this regard.

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CONCLUSIONS AND SIGNIFICANCE

A procedure to predict pressure drop and in situ configuration or holdup has been developed for horizontal gas liquid pipelines in which the liquid is in laminar flow and the gas in turbulent flow. The theoretical analysis solves the equations of motion for the liquid phase by including the interfacial stress caused by the turbulent gas interacting at the interface. The necessity for so doing is justified by the agreement of the model predictions with well over a hundred data points collected in pilot pipelines.

This experimental work was carried out with both air-water and air-glycerine solutions in 14-m. pipelines of diameter 0.0254, 0.0381, and 0.0508 m. Both pressure drop and holdup are difficult to measure accurately over the range of fluid rates which produce laminar liquid and turbulent gas flows, and procedures were developed to overcome this problem. Pressure drops were measured with recording transducers and the holdups were determined using in-line conductivity probes instead of the standard double valve trapping procedure.

A design procedure is presented which one can use with some confidence outside the range of pipe sizes and fluid properties used in this experimental work since no empirical fitting has been done to develop the models.

MODEL DEVELOPMENT

When two fluids in laminar motion flow in a pipeline, the less dense will flow above the more dense; and if both are Newtonian, the pressure drop and holdup can be predicted by solving the equations of motion in the following manner:

$$\mu_A \frac{\partial^2 v_A}{\partial x^2} + \mu_A \frac{\partial^2 v_A}{\partial y^2} = \frac{\partial P_A}{\partial z} \tag{1}$$

$$\mu_B \frac{\partial^2 v_B}{\partial x^2} + \mu_B \frac{\partial^2 v_B}{\partial y^2} = \frac{\partial P_B}{\partial z} \tag{2}$$

The following conditions are applicable:

The phase velocity at the wall is zero

$$v_A = 0 \qquad v_B = 0 \tag{3}$$

The phase pressure drops are equal

$$\frac{\partial P_A}{\partial z} = \frac{\partial P_B}{\partial z} = \frac{\Delta P_B}{L} = \frac{\Delta P_A}{L} \tag{4}$$

The stress and velocity at the interface are equal

$$\mu_A \frac{\partial v_A}{\partial y} = \mu_B \frac{\partial v_B}{\partial y} \tag{5}$$

$$v_A = v_B \tag{6}$$

These equations describe the physical situation shown in Figure 1.

The solution to this coupled set of equations yields velocity profiles v_A and v_B as a function of position in each phase. Volumetric flow rates Q_A and Q_B are obtained by integration of each velocity profile. The ratio of the flow rates can then be determined as a function of interface position for any two given fluids in stratified flow. Once the interface position is known, the holdups R_A and R_B are easily calculated. Since each equation for the volumetric flow rate contains the pressure drop, $\Delta P_B/L$ or $\Delta P_A/L$ may be computed once the interfacial position is known.

While the formulation of the problem is simple, the algebraic form of the solution is not, and a trial-and-error procedure must be followed if holdup and pressure drop are to be computed, given the input flow conditions Q_A, Q_B, μ_A, μ_B, ρ_A, ρ_B, and R. A useful solution to the laminar fluid-fluid problem has been presented in graphical form by Yu and Sparrow (1962), and their analysis allows us to establish the laminar-laminar region for any horizontal fluid-fluid system.

Although two fluids in laminar motion will flow in a stratified pattern, it is not obvious that such a pattern will be maintained if one of the fluids is in turbulent motion. Some insight into this problem is gained by using the Yu-Sparrow analysis to plot the laminar-laminar region on a flow-pattern chart after Baker (1961).

The laminar-laminar region for air-water flow in 1-in. and 4-in. pipe is shown on the Baker chart in Figure 2. (For this system λ and ψ are each equal to unity.) The following definitions are used for the phase Reynolds numbers, and the transition point has been taken as 2100.

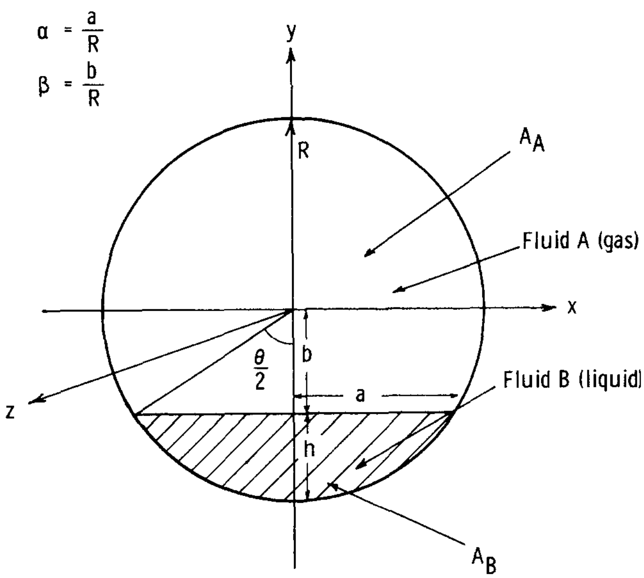

Fig. 1. Stratified pipe flow.

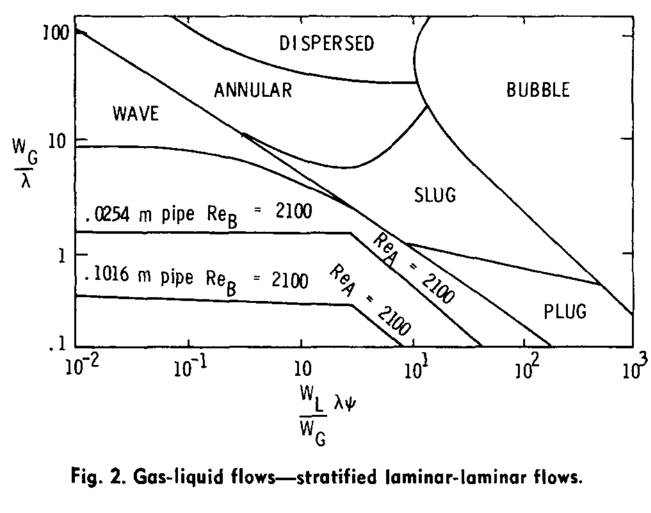

Fig. 2. Gas-liquid flows—stratified laminar-laminar flows.

$$Re_A = \frac{v_A H_A \rho_A}{\mu_A} \quad (7)$$

$$Re_B = \frac{v_B H_B \rho_B}{\mu_B} \quad (8)$$

$$v_A = \frac{Q_A}{R_A A} \qquad v_B = \frac{Q_B}{R_B A} \quad (9)$$

$$R_A = \frac{A_A}{\pi R^2} \qquad R_B = \frac{A_B}{\pi R^2} \quad (10)$$

$$A_A + A_B = \pi R^2 \quad (11)$$

$$H_A = \frac{4A_A}{p_A} \qquad H_B = \frac{4A_B}{p_B} \quad (12)$$

$$p_A = \pi D - R\theta + 2R \sin\frac{\theta}{2}; \qquad p_B = R\theta \quad (13)$$

A_A and A_B are shown in Figure 1. The gas is designated as the A fluid, and computations were performed using air at atmospheric pressure.

The Baker chart is not claimed to be precise in its designation of the boundaries between flow regimes but, even allowing for considerable error, stratified flow has been observed to exist when the air is in turbulent motion and the water laminar. This is the area lying above the laminar-laminar regions, Figure 2, having as its approximate right-hand boundary a straight line extension of the line $Re_A = 2100$.

For laminar-liquid/turbulent-gas flows, a mathematical description can be developed that uses Equation (2) and well-established procedures for dealing with turbulent incompressible gas flows. When a gas in turbulent motion flows above a laminar flowing liquid, the boundary conditions needed to solve Equation (2) are as follows:

$$v_B = 0 \quad \text{at the wall} \quad (14)$$

$$\mu_B \frac{\partial v_B}{\partial y} = \tau_s \quad \text{at the gas-liquid interface} \quad (15)$$

The shear stress in the gas phase is represented by τ_s which is related to $\Delta P_A/L$ as follows:

$$\tau_s = \frac{\Delta P_A}{L} \frac{H_A}{4} \quad (16)$$

For this relationship to hold, we must assume that the gas sees a solid boundary at the liquid interface. This assumption is shown to be sound by Jensen (1972) and Weldy (1971).

Equation (2) with Conditions (14) and (15) can be solved numerically and the results presented graphically. In addition, an approximate analytical result will be discussed to illustrate the procedure more clearly and to provide an equation useful for preliminary design estimates.

An analytical solution to Equation (2) is obtained by assuming that

$$\frac{\partial^2 v_B}{\partial x^2} = C \frac{\partial^2 v_B}{\partial y^2} \quad (17)$$

With this assumption, Equation (2) can be rewritten

$$\frac{\partial^2 v_B}{\partial y^2} = C_B K \quad (18)$$

where

$$K = \frac{1}{\mu_B} \frac{\Delta P_B}{L} \quad (19)$$

and

$$C_B = \frac{1}{1 + C}$$

Although the algebra is tedious because of the pipe geometry, Equation (18) can be solved to yield an expression relating volumetric flow rate Q_B to pressure drop $\Delta P_B/L$, the fluid properties, and the position of the interface in the pipe.

$$\frac{Q_B}{KR^4} = -\frac{C_B}{12}\Big[2\alpha^3\beta - 15\alpha\beta + (3 + 12\beta^2)\sin^{-1}\alpha + \frac{\tau_s}{\mu_B K C_B R}(12\alpha - 4\alpha^3 - 12\beta\sin^{-1}\alpha)\Big] \quad (20)$$

The terms α and β are defined in Figure 2.

We can eliminate τ_s from this expression by using the relationship developed as Equation (16). The dimensionless term

$$Q^* = \frac{Q_B}{8KR^4} \quad (21)$$

is introduced to modify Equation (20) to the following form:

$$Q^* = -\frac{C_B}{96}\Big[(2\alpha^3\beta - 15\alpha\beta + [3 + 12\beta^2]\sin^{-1}\alpha) + \frac{H_A}{D}(6\alpha - 2\alpha^3 - 6\beta\sin^{-1}\alpha)\Big] \quad (22)$$

The manipulations leading to Equation (22) are similar to those outlined by Buffham (1968) in his analysis of open-channel flows. Buffham assumed that $C_B = 1$ in his approximate analytical analysis, which is equivalent to the following approximation:

$$\frac{\partial^2 v_B}{\partial y^2} >> \frac{\partial^2 v_B}{\partial x^2} \quad (23)$$

Buffham's equation doesn't contain the second term on the right-hand side of Equation (22) since this is the term accounting for interfacial shear. Except for this and an unfortunate typographical error in the Buffham paper, his analysis agrees with Equation (22).

Equation (22) provides a means of computing pressure drop and holdup if fluid flow rate, pipe size, and fluid properties are specified and if C_B is either known as a function of interfacial position or assumed to be a constant as in the Buffham analysis. The iterative calculation procedure required is given below.

1. A value of h/D is assumed, α and β are computed from any standard math table that gives the geometrical relationships for circular segments (Figure 2).
2. A numerical value is obtained for Q^* using Equation (22). Since Q_B and μ_B are known, $\Delta P_B/L$ can be calculated [Equation (21)].
3. The gas-phase pressure drop $\Delta P_A/L$ must be equal to $\Delta P_B/L$. This is checked by computing $\Delta P_A/L$ using established single-phase flow procedures.

$$\frac{\Delta P_A}{L} = \frac{2 f_A v_A^2 \rho_A}{H_A} \quad (24)$$

$$H_A = \frac{4A_A}{p_A} \quad (12)$$

The gas-phase friction factor f_A can be obtained graphically or by using the Blasius equation

$$f_A = 0.079\, Re_A^{-0.25}$$

For rough preliminary estimates, it is sometimes more convenient simply to use $f_A = 0.005$.

4. If $\Delta P_A/L$ is not equal to $\Delta P_B/L$, a new value of h/D is selected and the procedure repeated until a satisfactory

match is obtained.

If the flow situation is such that h/D is less than 0.1, Equation (22) can be used with $C_B = 1$. This approximation becomes increasingly less valid as h/D increases. In fact, when the pipe is half full of liquid, C_B has a value close to 0.25.

If the approximation represented as Equation (23) is not made, an expression for Q^* is most conveniently found by numerical integration of Equation (2) using the boundary conditions represented by Equation (15). Jensen (1972) presents such a solution, and his results are shown in Table 1 under the heading Q^*LLS. A graphical representation is shown in Figure 3 as Q^* vs. h/D. Shown on the same figure is the solution to the gas-liquid flow problem using Buffham's free surface analysis to describe the liquid flow. A comparison of the two curves shows the effect of the boundary condition at the gas-liquid interface [Equation (15)] on the Q^* vs. h/D relationship.

The procedure described above to obtain holdup and pressure drop can be equally well used with the curve shown in Figure 3 to obtain Q^* once an initial value of h/D is selected to begin the iterative process. Alternatively, the following approximate expression for C_B can be used with Equation (22) to avoid the graphical procedure:

TABLE 1. LAMINAR-LIQUID TURBULENT-GAS FLOWS

$Q^*LLS = \dfrac{Q_B}{8KR^4}$ [From numerical solution of Equation (2)]	h/D
1.58×10^{-4}	0.05
8.87×10^{-4}	0.10
4.89×10^{-3}	0.20
1.28×10^{-2}	0.30
2.40×10^{-2}	0.40
3.73×10^{-2}	0.50
5.08×10^{-2}	0.60
6.15×10^{-2}	0.70
6.66×10^{-2}	0.80
6.31×10^{-2}	0.90

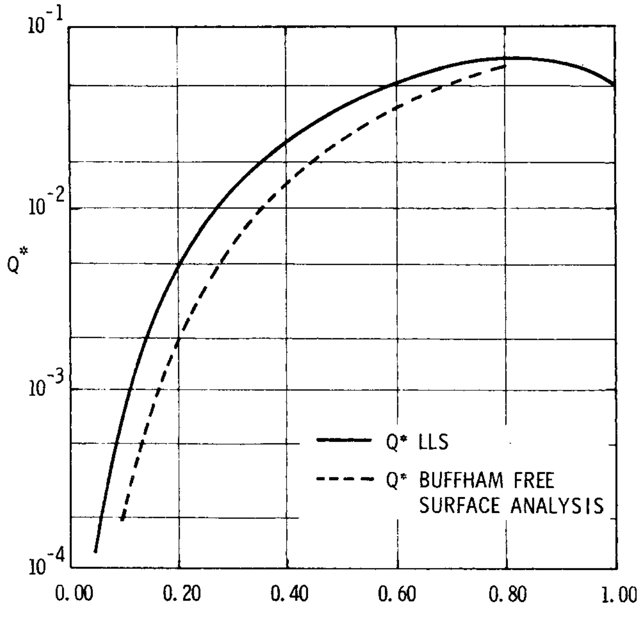

Fig. 3. Pressure drop/holdup correlation for stratified gas-liquid flows.

$$C_B = 1.05 \exp\left[-1.46\frac{h}{D}\right] \tag{25}$$

This equation was obtained by comparing the numerical solution and Equation (22) to obtain C_B as a simple function of h/D.

COMPARISON WITH EXPERIMENT*

Figure 2 shows the approximate range of fluid flow rates that will produce stratified flow in horizontal pipes. To completely check the predictions of the procedure developed, both pressure drop and holdup measurements should be made over the entire range of gas and liquid flow rates where there is turbulent gas and laminar liquid flow. Extrapolation of single-phase experience tells us that liquid viscosity is by far the most important physical property to vary, and at least one liquid other than water should be used in the experimental program. Varying pipeline diameters should also be used; however, it is difficult to go to very large diameter pipes because of the fluid pumping limitations in most laboratories.

It is not a trivial matter to obtain the needed data. Pressure drops are very small over the usual length of pilot pipelines (about 14 m.), and unless one uses special techniques, experimental error or random fluctuations can constitute a high percentage of the measured value, making the data worthless. It is even more difficult to obtain accurate holdup measurements. The usual procedure of trapping the two-phase mixture in the pipe and then measuring liquid volume leads to experimental error which in some cases, as with pressure drop data, are a high percentage of experimental value.

These difficulties were overcome in a series of experiments by Arruda (1970) and Jensen (1972) who covered the complete range of gas and liquid flow rates that produced stratified flow in 0.0254-, 0.0381-, and 0.0508-m. diameter pipelines. They collected well over a hundred data points using air-water and air-glycerine solutions in the 14-m. pilot pipelines at the University of Delaware.

Air flows were measured by a calibrated orifice and liquid flows by calibrated rotameters. Pressure drops were measured at taps in the pipelines four meters on either side of the pipeline center, by a differential pressure transducer (Whitaker P 109 D and CD 25) and recorded (Honeywell Visicorder Model 1508A). Liquid levels were measured by a micrometer and dip stick and by an electric depth probe similar to that described by Lamb et al. (1960). The probe consisted of two double wire conductivity probes in the pipe, one vertical for measuring and one lying flat on the bottom to act as a reference. These probes were calibrated with static water levels and with flowing water using the dip stick and a cathetometer. Comparison of pressure drop data using a transducer to previously obtained inclined manometer data (Arruda, 1970) showed that greater reliability and reproducibility was obtained with the transducer. The recorded depth measurements gave a graphical representation of the interface and were used as a criteria for transitions. For further details refer to Jensen (1972). The range of variables explored is given in Table 2. Since the theory predicts a significant viscosity effect, this variable was varied over a range of two orders of magnitude.

In Figure 4, all data are plotted as Q^* vs. h/D. This was done by computing Q^* using Equation (21) and the

* Tables of experimental data for both the water and glycerine solution tests have been deposited as Document No. 02392 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 305 E. 46 St., New York, N. Y. 10017 and may be obtained for $1.50 for microfiche or $5.00 for photocopies.

TABLE 2. RANGE OF VARIABLES STUDIED

Gas flow	2.36×10^{-3}	to	8.96×10^{-3}	m^3/s
Liquid flow	1.89×10^{-6}	to	1.89×10^{-5}	m^3/s
Pipe diameters	2.54×10^{-2}	to	5.08×10^{-2}	m
Viscosity gas	1.86×10^{-5}			$N\ s/m^2$
liquid	1.00×10^{-3}	to	3.00×10^{-1}	$N \cdot s/m^2$
Density gas	1.22			Kg/m^3
liquid	1,000			Kg/m^3
Pressure drop/length	16	to	65	N/m^3
Holdup, R_A	0.01	to	0.55	

measured values for Q_B and $K = \frac{1}{\mu_B}\frac{\Delta P_B}{L}$. The value of h/D was computed using the experimentally determined value of liquid holdup R_A and the simple geometrical relationships. Figure 4 thus compares the combined effect of pressure drop and holdup with the theoretical development presented as Figure 3. The comparison of these two elements separately is shown in Figures 5 and 6. As can be seen from these figures, agreement with theory is good for pressure drop and for the combined effect of pressure drop and holdup, while being adequate for holdup alone. The greater scatter in the holdup plot reflects the experimental difficulties associated with making this measurement.

COMPARISON WITH OTHER PROCEDURES

Figure 7 shows Q^* vs. h/D for the theory developed in this work (Q^*LLS), the method recently developed by Agrawal et al. (1973) and the widely used Lockhart and Martinelli (1949) correlation. Comparison with Figure 4 which has the data points shows that the Q^*LLS model is superior to both the other models in its capability to predict experimental behavior. The Agrawal et al. (1973) model is particularly poor at values of h/D below 0.2. The Lockhart and Martinelli (1949) model predicts lower values of Q^* than that observed over the complete range of h/D, as does the Agrawal et al. (1973) model at values of h/D over 0.30.

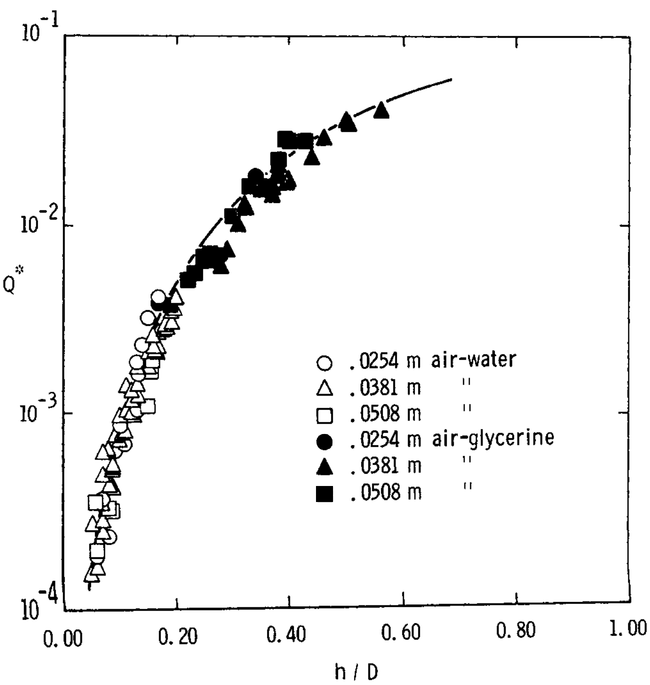

Fig. 4. Comparison of holdup and pressure drop data with prediction.

DESIGN PROCEDURE

An essential part of any design problem involving a two-phase tubular pipe is the capability to predict pressure drop and in situ configuration given fluid-flow rates, fluid

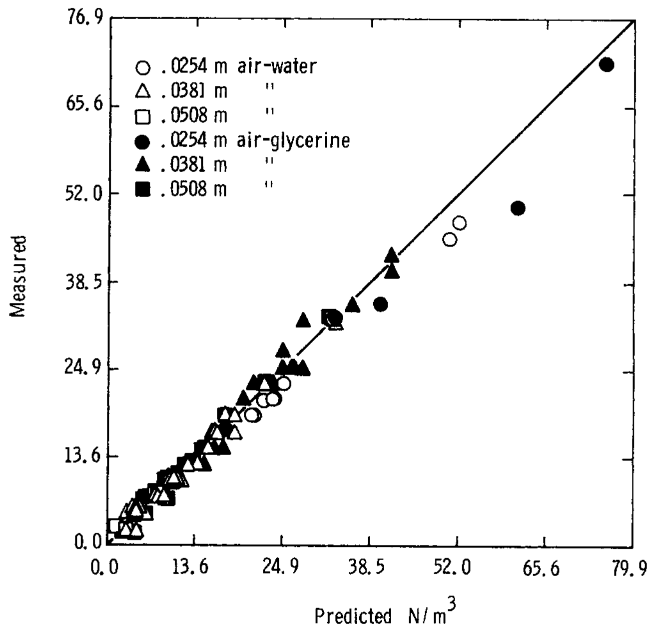

Fig. 5. Predicted and measured pressure drop.

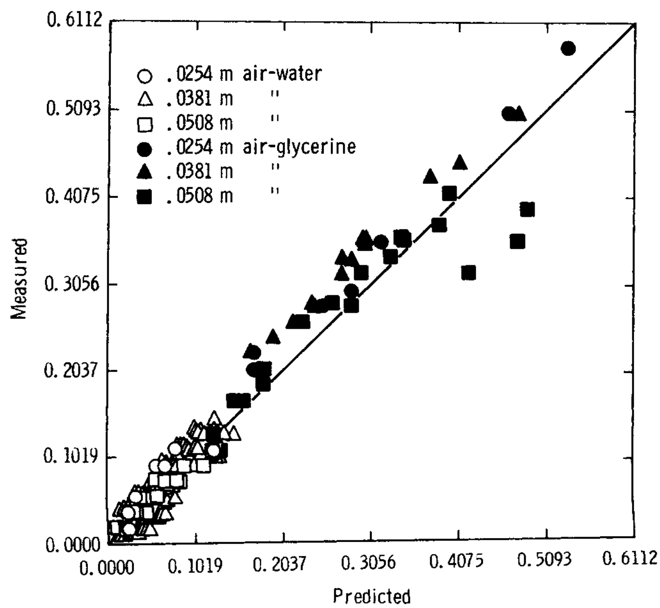

Fig. 6. Predicted and measured holdup.

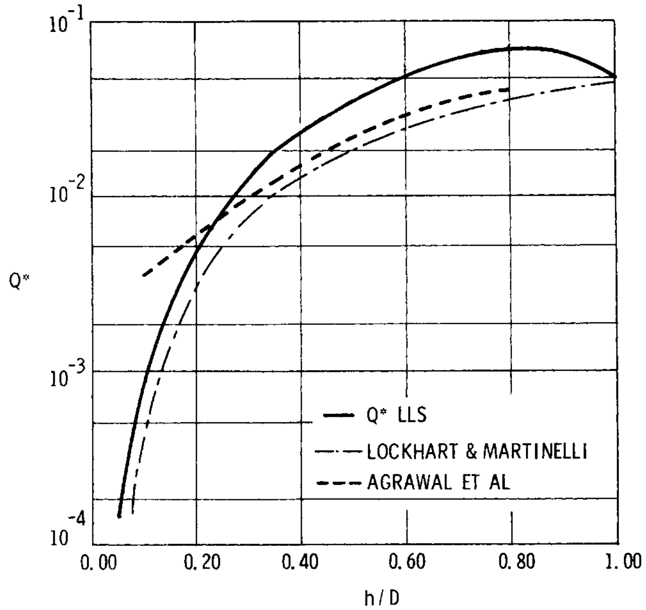

Fig. 7. Comparison with existing correlations.

properties, and pipe geometry. The following procedure is most conveniently carried out with a computer. A sophisticated program has been developed by Kimpel (1970). Here we outline the essential logic, and the reader may compute by hand or set up his own program.

1. Convert raw input data to a form in which Q_A, Q_B, μ_A, μ_B, ρ_A, ρ_B, and D are in a consistent set of units.
2. To use the procedure outlined in this paper it is necessary to determine if a laminar liquid and turbulent gas flow exists.

(i) First compute the superficial Reynolds number for each phase using Equations (7) and (8) with D, the pipe diameter, in place of H_A and H_B. Compute v_A and v_B as if each fluid occupied the entire pipe cross-section.

(ii) If the superficial Reynolds numbers are both less than 2100, use the Yu and Sparrow (1962) analysis to determine h/D and the pressure drop $\Delta P_B/L = \Delta P_A/L$. Recompute the Reynolds number using Equations (7) and (8) as written to check for laminar-laminar flows.

(iii) If the superficial liquid phase Reynolds number is less than 2100 and if the superficial gas phase Reynolds number is greater than 2100, then the iterative computation described in the model development section of this paper should be followed. The Reynolds number must be recomputed using Equations (7) and (8) to check on the nature of the flow field.

3. If the superficial Reynolds numbers are well in excess of 2100 for both phases one must resort to some of the standard correlations available. These are discussed in the references listed in the Scope section of this paper. A system which has superficial Reynolds numbers within a factor of two of the values for turbulent-gas, laminar-liquid flows should be checked by following step 2(iii) and calculating Reynolds numbers as defined by Equations (7) and (8).

NOTATION

A_i = area of pipe occupied by phase i
C_B = flow constant for Buffham's solution
D = pipe diameter
f = Fanning friction factor
g = acceleration due to gravity
h = height of liquid in pipe
H_i = hydraulic radius of phase i
K = $\Delta P_B/\mu_B L$
L = pipe length
p_i = wetted perimeter of phase i
P_i = pressure in phase i
Q_i = volumetric flow rate of phase i
Q^* = dimensionless flow rate defined by Equation (21)
R_i = holdup of phase i
R = pipe radius
Re_i = Reynolds number of phase i
v_i = average linear velocity of phase i
W = mass velocity, $K_g/m^2 \cdot$ sec. Figure 2
x, y, z = coordinates defined on Figure 1
α, β defined on Figure 1
μ_i = viscosity phase i
ρ_i = density phase i
τ_s = stress at the interface
χ = $[(\rho_g/1.2014)(\rho_L/997.95)]^{1/2}$
ψ = $\left[\left(\frac{73000}{\sigma}\right)\left[\frac{\mu}{1000}\right]\left(\frac{997.95}{\rho_L}\right)\right]^{1/3}$

Subscripts

A = upper phase
B = lower phase
G = gas phase, Figure 2
L = liquid phase, Figure 2

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